Lesson 35 - Double Integrals - Average Value

I. Formula

II Examples

Quiz 12 due 7:41 AM.

Final Exam

Posted Nov 14, 2023 2:06 PM

The final exam will be on Friday, December 15 from 10:30am-12:30pm.

A memo with detailed information will be posted in Brightspace closer to the exam.

Plan travel arrangements accordingly. No final exams will be given early.

I Average Value Formulas

- Ave value of $x_1, x_2, \ldots, x_n$

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- Ave value of $f(x)$ on $[a,b]$

$$f_{ave} = \frac{\int_a^b f(x)\,dx}{b-a} = \frac{F(b)-F(a)}{b-a}$$

length of interval 1-D

- Ave value of $f(x,y)$ on a bounded region R

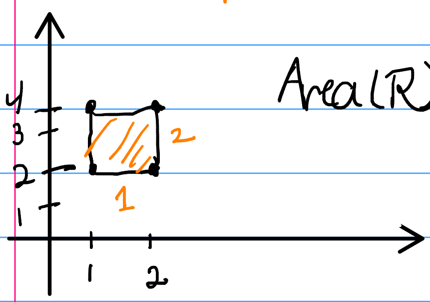
$$f_{ave} = \frac{\iint_R f(x,y)\,dA}{\text{Area}(R)}$$

dy dx or dx dy

II. Examples

$\boxed{\text{Ex}}$ (Based on Rogawski et. al. §15.1 #32)

Find the ave. value of $f(x,y) = e^{3x-y}$ on the $\underbrace{\text{rectangle}}_{R}$ with vertices $(1,2)\,(1,4)\,(2,4)$ and $(2,2)$.



$\text{Area}(R) = 1 \times 2 = 2$

$$f_{ave} = \frac{\int_1^2 \int_2^4 e^{3x-y}\, dy\, dx}{}$$

top: $y=4$ right: $x=2$

bottom: $y=2$ left: $x=1$

$$= \frac{1}{2} \int_1^2 \int_2^4 e^{3x} e^{-y}\, dy\, dx$$

$$= \frac{1}{2} \int_1^2 \left[e^{3x}(-e^{-y}) \Big|_{y=2}^{y=4} \right] dx$$

$$= \frac{1}{2} \int_1^2 \left[-e^{-4} e^{3x} + e^{-2} e^{3x} \right] dx$$

$$= \frac{1}{2} \left[-\frac{1}{3} e^{-4} e^{3x} + \frac{1}{3} e^{-2} e^{3x} \right] \Big|_{x=1}^{x=2}$$

$$= \frac{1}{6} e^{3x} \left(e^{-2} - e^{-4} \right) \Big|_{x=1}^{x=2}$$

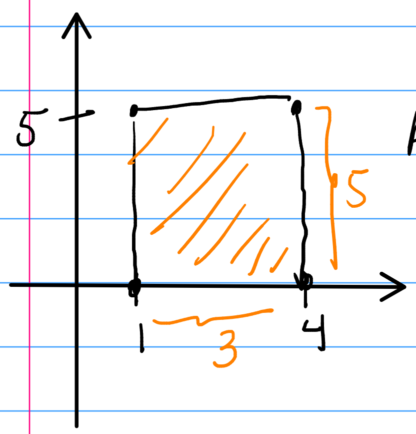
$$= \frac{1}{6} (e^{-2} - e^{-4}) \left[e^{6} - e^{3} \right]$$

$$\approx 7.4764$$

$\boxed{\text{Ex}}$ (Based on Rogawski et.al. §15.1 #33)

Find the average value of $f(x,y) = \dfrac{1}{\sqrt{x+y}}$

on the rectangle with vertices $(1,0)$ $(1,5)$ $(4,5)$ $(4,0)$.



$\text{Area}(R) = 3 \times 5 = 15$

top: $y=5$ right: $x=4$
bottom: $y=0$ left: $x=1$

$$f_{ave} = \frac{1}{15}\int_1^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx$$

$(x+y)^{-1/2}$

$$= \frac{1}{15}\int_1^4 \left[2(x+y)^{1/2}\Big|_{y=0}^{y=5}\right]dx$$

$$= \frac{1}{15}\int_1^4 \left[2(x+5)^{1/2} - 2x^{1/2}\right]dx$$

$$= \frac{1}{15}\left[2\cdot\frac{2}{3}(x+5)^{3/2} - 2\cdot\frac{2}{3}x^{3/2}\Big|_{x=1}^{x=4}\right]$$

$$= \frac{4}{45}\left[\left((9)^{3/2} - (4)^{3/2}\right) - \left((6)^{3/2} - (1)^{3/2}\right)\right]$$

$$= \frac{4}{45}\left[(27-8) - \left((\sqrt{6})^3 - 1\right)\right]$$

$$\approx 0.47138$$

Side note

$u = 2+y$
$du = dy$

$$\int (2+y)^{-1/2}\,dy$$
$$= \int u^{-1/2}\,du$$
$$= 2u^{1/2} + C$$
$$= 2(2+y)^{1/2} + C$$

Monday: Questions about LON-CAPA "After Exam 3" Review